Integrating Crew Scheduling and Crew Rostering for rail freight with train delays

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November 18th, 2025

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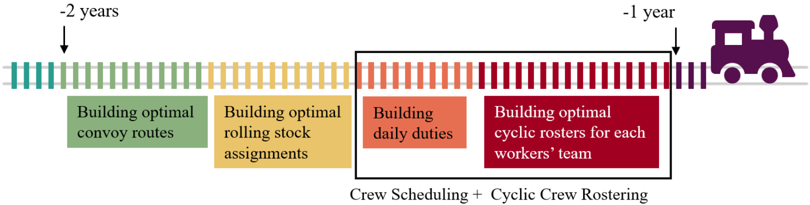
Rail freight in France

- Modal share for rail freight: 18% in Europe (10% in France)
- Between 1800 and 2000 trains per week
- Many differences with passengers transportation:
  - Priority goes to passengers
  - Trains mostly at night

Resource planning



Contributions

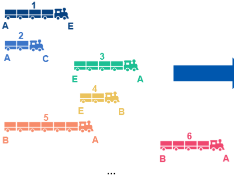
We propose a complete methodology:

- modeling the "resilience" of rosters to train delays at the SNCF
- solving the integrated problem while handling train delays without sampling
- providing a tight lower bound
- leading to an average 8% cost reduction for the deterministic version

Problem definition

Input: Trains on a typical week

Output: Covering of trains by "blocks" with minimum cost, each block assigned to a team (each block then placed in a cyclic roster)



Team A:

sun	mon	tue
duty 7	duty 1	on-call
10h-17h	12h-21h	
A - A	A - A	

sat	sun	mon	tue
duty 5	duty 6	duty 2	duty 3
11h-17h	5h-16h	12h-19h	5h-11h
A - E	E - A	A - B	B - A

Team B:

mon	tue	wed
duty 4	duty 8	duty 9
12h-20h	11h-17h	5h-12h
B - B	B - B	E - B

cost = 10

Team A:

mon	tue	wed	thu	fri	sat	sun
duty 1	on-call	R	R	R	duty 5	duty 6
12h-21h					11h-17h	5h-16h
A - A					A - E	E - A
duty 2	duty 3	R	R	R	R	duty 7
12h-19h	5h-11h					10h-17h
A - B	B - A					A - A

Team B:

mon	tue	wed	thu	fri	sat	sun
duty 4	duty 8	duty 9	R	R	R	R
12h-20h	11h-17h	5h-12h				
B - B	B - B	E - B				

Stochastic train delays: context

Train departure and arrival delays (primary delays and secondary delays) $\rightarrow$ disruptions of rosters with propagation within duties.

When delays exceed some threshold, disruptions occur and they induce prohibitive adjustments:

- a full duty, or some of its trains, must be operated by backup drivers;
- an on-call must be added within a block or at its end.

Stochastic train delays: fragility and assumptions

Assumptions:

- primary delays are independent variables (not necessarily identically distributed),
- their distributions are known,
- their support is finite.

H: random variable for arrival times (with primary and secondary delays)
$\sigma(h, b)$: number of disruptions in the realization $H = h$.

Roster "fragility" score:

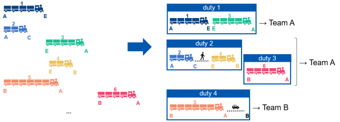
$$\text{fragility}(r) := \mathbb{E}\Big[\sum_{b \in r} \sigma(H, b)\Big]$$

Heuristic: sequential approach

❶ Crew Scheduling

Input: Trains on a typical week

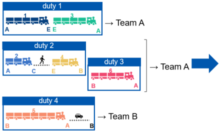
Output: Covering of trains by daily duties with minimum cost, each duty assigned to a team



❷ Crew Rostering

Input: Duties, each assigned to a team

Output: Covering of trains by blocks with minimum cost, each block assigned to a team (then placed in a roster)



Team A:

sat	sun	mon	tue
on-call	duty 7 5h-14h A - A	duty 1 12h-21h A - A	on-call

sat	sun	mon	tue
duty 5	duty 6	duty 2	duty 3
11h-17h	11h-18h	12h-19h	5h-11h
A - E	E - A	A - B	B - A

Team B:

mon	tue	wed
duty 4	duty 8	duty 9
12h-20h	11h-17h	5h-12h
B - B	B - B	E - B

cost = 11

→ Leads to sub-optimal solutions

→ Hard to account optimally for uncertainties

Model

$$\begin{aligned} \text{minimize} \quad & \sum_{i\in I}\sum_{b\in B_i} c_b x_{b,i} \\ \text{s.t.} \quad & \sum_{i\in I}\sum_{b\in B_i : t\in b} x_{b,i} \geqslant 1 & \forall t \in T \\ & \sum_{i\in I}\sum_{b\in B_i} \mathbb{E}[\sigma(H,b)] x_{b,i} \leqslant S \\ & x_{b,i} \in \{0,1\} & \forall i \in I, \forall b \in B_i \end{aligned}$$

B_i = set of feasible "blocks" for the team i

$\rightarrow$ combinatorial explosion

National input:

1800 trains $\rightarrow \sim 10^{25}$ blocks

Column Generation: standard resolution methodology of a linear program when the number of variables is large

Column Generation

$$\begin{aligned}
\text{minimize} \quad & \sum_{i\in I}\sum_{b\in\overline{B}_i} c_b x_{b,i} \\
\text{s.t.} \quad & \sum_{i\in I}\sum_{b\in\overline{B}_i : t\in b} x_{b,i} \geqslant 1 \qquad \forall t\in T \\
& \sum_{i\in I}\sum_{b\in\overline{B}_i} \mathbb{E}[\sigma(H,b)] x_{b,i} \leqslant S \\
& x_{b,i}\in\{0,1\} \qquad \forall i\in I, \forall b\in\overline{B}_i\subset B_i
\end{aligned}$$

Solving the linear relaxation with B_i:

- Iteratively
  - solving the linear relaxation with $\overline{B}_i$
  - retrieving dual information
  - searching for solutions to a "pricing sub-problem"
  - adding elements to $\overline{B}_i$
- Optimality guarantee when no elements can be added

Solving the integer problem with B_i:

- Upper bound and feasible solution with $\overline{B}_i \subseteq B_i$

Challenge: Solving the pricing sub-problem quickly

For our problem:

Pricing sub-problem = Shortest path with constraints (Resource Constrained Shortest Path)

Resource Constrained Shortest Path: setting

Input: Graph with vertices o and d, feasible $o-d$ paths $\mathcal{P}_{od}$ and a (non-linear) cost function c
Ouput: Feasible $o-d$ path $P \in \mathcal{P}_{od}$ with minimum cost $c(P)$

Resources: Vector tracking accumulated valuable quantities along a path through an extension (non-linear) function

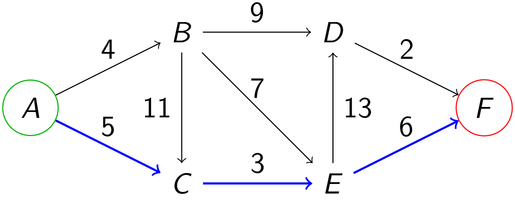
e.g.: shortest $o-d$ path under time budget
$\rightarrow$ *resources: accumulated length, accumulated arc time*
e.g.: shortest $o-d$ path under stochastic time budget
$\rightarrow$ *resources: accumulated length, accumulated arc time distribution*

Shortest path

Usual shortest path

Input: Graph with vertices o and d, non-negative cost on each arc

Ouput: $o - d$ path with minimum cost



$\rightarrow$ Dijkstra's algorithm

$\rightarrow$ A^* search algorithm

Shortest path

A^* search algorithm

Principle: Enumeration algorithm with bounds to discard paths

- Bound b_v under-estimating cost of shortest path from any vertex v to d
- Discard paths P from o to v with "estimated cost" $c(P) + b_v$ greater than one of an explored $o - d$ path

$\rightarrow$ with $b_v = 0$: Dijkstra's algorithm

e.g.: shortest route on a map
$\rightarrow b_v =$ *distance as the crow flies*

Resource Constrained Shortest Path: algorithms

Resolution: Enumeration algorithm using

- Key: order of paths processing
- Bound: under-estimate of the resources and cost to reach d
- [not in A^*] Dominance: comparison of resources for paths with same cost

Different algorithms:[1]

- Generalized A^*

  Key = "estimated cost", discard paths using "estimated cost"
- Label dominance

  Key = cost of path, discard paths using dominance
- Label correcting

  Key = "estimated cost", discard paths using "estimated cost" and dominance

[1] synthetized by A.Parmentier in Algorithms for Non-Linear and Stochastic Resource Constrained Shortest Paths, 2017.

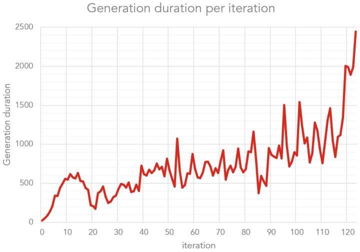
For our problem

How to handle the duty and block feasibility constraints and the stochastic constraint?

- One graph per team, with trains as vertices and arcs encoding possible successions of trains.
- Resources keep track of several indicators, as:
  - block indicators (number of days, etc.)
  - duties indicators (range, driving duration, etc.)
  - delay distributions

Results: without train delays

	Sequential approach		Our approach		
	Objective	Time	Obj. lower bound	Obj. upper bound[2]	Total time
Instance 280 trains	228	3min	205.3	**209 (-8%)**	21h07min
Instance 925 trains	832	26min	711.9	**730 (-12%)**	23h48min
Instance 1810 trains	1633	1h04min	1395.0	**1494 (-8%)**	40h16min



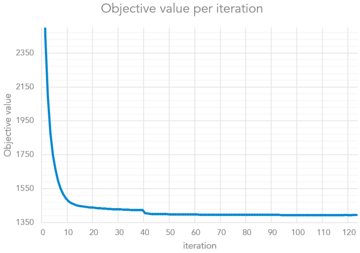


Figure: Column Generation indicators per iteration

[2] solving MILP with CG output variables and a 1h time limit

Next goals

- Gap closing (or strenghtening) for large instances without train delays
- Getting results with train delays on small and large instances
- Challenging the finite support assumption on the distributions

Thank you!