

Integrated crew management for rail freight

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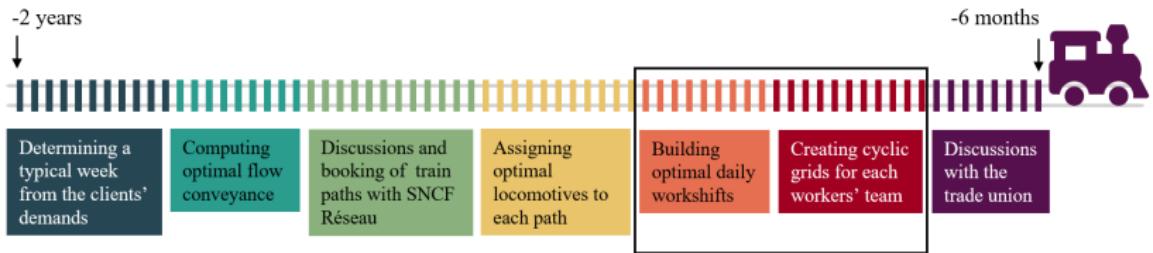
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Rail freight in France

- 18% of rail freight in Europe (10% in France)
- Between 1800 and 2000 trains per week
- Many differences with passengers transportation...
 - Priority goes to passengers
 - Trains mostly at night
 - Client satisfaction: delivery on time
 - Key goal: reducing operational costs

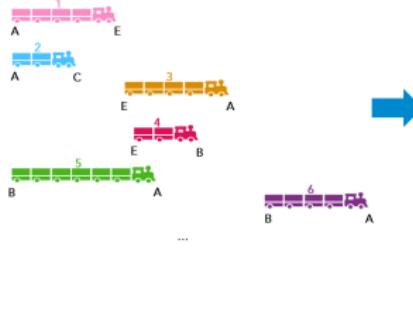
Resource planning



Problem definition

Input: Trains on a typical week

Output: Covering of trains by “blocks” with minimum cost, each block assigned to a team (each block then placed in a grid)



Team A:

sat	sun	mon	tue
WS 5	WS 6	WS 2	WS 3
11h 17h	8h 15h	12h 19h	5h 11h
A E	E A	A B	B A
sun	mon	tue	
WS 7	WS 1	d	
10h 17h	12h 21h		
A A	A A		

Team B:

mon	tue	wed
WS 4	WS 8	WS 9
12h 20h	11h 17h	5h 12h
B B	B E	E B

cost = 10

Team A:

mon	tue	wed	thu	fri	sat	sun
WS 1					WS 5	WS 6
12h 21h	d		R	R	R	A E
A A					R	E A
WS 2	WS 3				R	WS 7
12h 19h	5h 11h		R	R	R	10h 17h
A B	B A					A A

Team B:

mon	tue	wed	thu	fri	sat	sun
WS 4	WS 8	WS 9				
12h 20h	11h 17h	5h 12h	R	R	R	R
B B	B E	E B				

Heuristic: sequential approach

1 Building shifts from trains

Input: Trains on a typical week

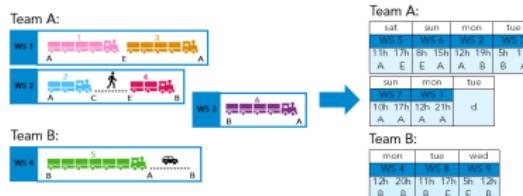
Output: Covering of trains by shifts with minimum cost, each shift assigned to a team



2 Building grids from shifts

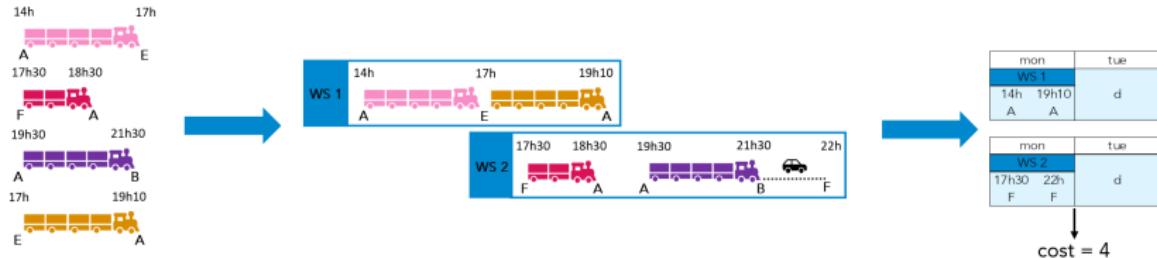
Input: Shifts, each assigned to a team (*output above*)

Output: For each team, covering of shifts by blocks with minimum cost

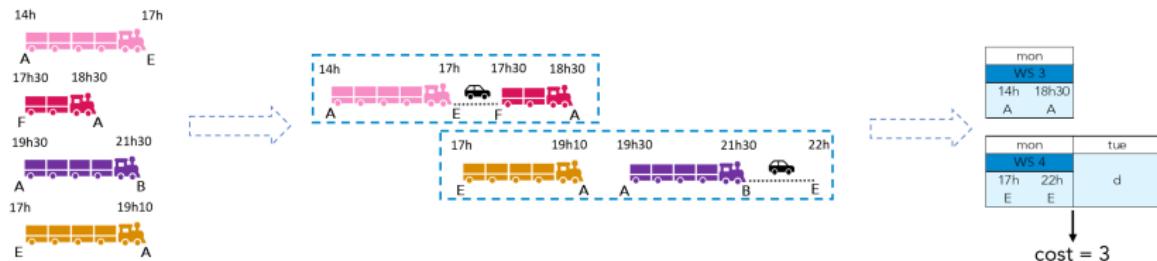


Sub-optimal approach

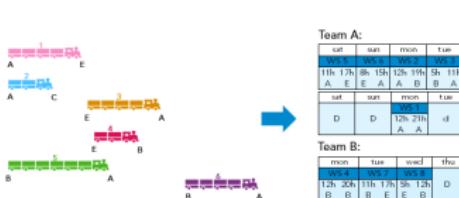
Sequential approach:



Optimal approach:



Model



$$\begin{aligned}
 & \text{Min} \sum_{i \in I} \sum_{b \in B_i} c_b x_{b,i} \\
 & \text{s.t.} \quad \sum_{i \in I} \sum_{\substack{b \in B_i \\ t \in b}} x_{b,i} \geq 1 \quad \forall t \in T \\
 & \quad x_{b,i} \in \{0, 1\} \quad \forall i \in I, \forall b \in B_i
 \end{aligned}$$

B_i = set of feasible “blocks” for the team i

→ combinatorial explosion

National input:

1 800 trains → 200 000 shifts → $\sim 10^{25}$ blocks

Column Generation: standard resolution methodology of a **linear program** when the number of variables is large

Column Generation

Generation of variables throughout the resolution → solving the pricing sub-problem

Key: Solving the pricing sub-problem **quickly**

For our problem:

Pricing sub-problem = Shortest path with constraints (Resource Constrained Shortest Path)

→ **NP-hard** problem

Shortest path with constraints

Input: Graph with vertices o and d , cost function and feasibility function on each path

Output: Feasible $o - d$ path with minimum cost

Feasibility is expressed as: “resources” accumulated along the path are under a given threshold

ex.: *shortest $o - d$ path under energy budget*

→ *resources: accumulated arc energy*

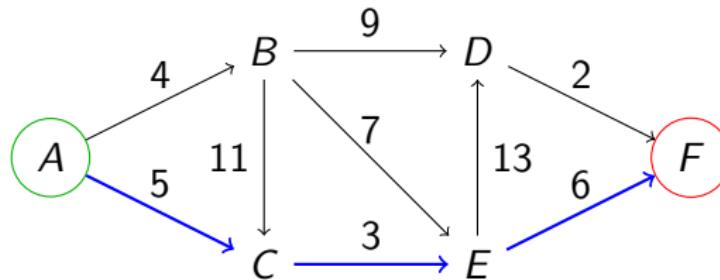
minimum-cost block under shifts and block feasibility constraints

→ *resources: accumulated shift range, number of days in block, etc.*

Shortest path

Usual shortest path

Input: Graph with vertices o and d , non-negative cost on each arc
Output: $o - d$ path with minimum cost



→ Dijkstra's algorithm
→ A^* search algorithm

Shortest path

*A** search algorithm

Principle: Enumeration algorithm with bounds to discard paths

- Bound b_v under-estimating cost of shortest path from any vertex v to d
- Discard paths P from o to v with “estimated cost” $c(P) + b_v$ greater than one of an explored $o - d$ path

→ with $b_v = 0$: Dijkstra’s algorithm

ex.: *shortest route on a map*
→ $b_v = \text{distance as the crow flies}$



Shortest path with constraints

Resolution: Enumeration algorithm using

- Key: order of paths processing
- Bound: under-estimate of the resources and cost to reach d
- [not in A^*] Dominance: comparison of resources for paths with same cost

Different algorithms¹:

- Generalized A^*

Key = “estimated cost”, discard paths using “estimated cost”

- Label dominance

Key = cost of path, discard paths using dominance

- Label correcting

Key = “estimated cost”, discard paths using “estimated cost” and dominance

¹ synthetized by A.Parmentier in Algorithms for Non-Linear and Stochastic Resource Constrained Shortest Paths, 2017.

For our problem

- **Vertices** are trains
- **Arcs** can be of different types, mainly:
 - succession of trains within shift
 - change of shifts between trains
- **Resource** contains several indicators, as:
 - block indicators (number of days, etc.)
 - previous shift indicators (range, driving duration, etc.)
 - current shift indicators

Results

	Sequential approach		Our approach				
	Objective	Time	Obj. lower bound	Obj. upper bound ²	Total time	Nb iterations	Nb variables
Instance 195 trains	176	2s	149.9	151 (-14%)	15min31	8	128 527
Instance 265 trains	261	3s	227.3	229 (-12%)	28min13	11	175 716
Instance 400 trains	402	5s	344.2	347 (-13%)	32min46	9	108 150

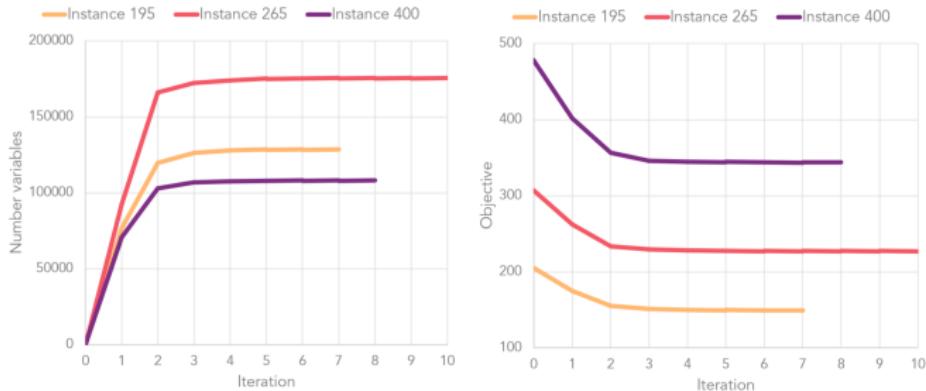


Figure: Column Generation indicators per iteration

²solving MILP with CG output variables

Current limitations

- Overall computation time
 - Preprocessing time: building pricing graph
 - Pricing sub-problem time
- Extend results to a national instance

Thank you!